Dark Energy in Extra Dimensions and String Theory: Consistency Conditions*

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Abstract. The smallness of the cosmological constant is one of the basic problems in particle physics and cosmology. Various attempts have been made to explain this mystery, but no satisfactory solution has been found yet. The appearance of extra dimensions in the framework of brane world systems seems to provide some new ideas to address this problem form a different point of view. We shall discuss some of these new approaches and see whether or not they lead to an improvement of the situation. We shall conclude that we are still far from a solution of the problem.

1 Introduction

We know that the cosmological constant is much smaller than one would naively expect. This led to the belief that a natural approach to this problem would be a mechanism that explains a vanishing value of this vacuum energy. While cosmological observations[1,2] seem to be consistent with a nonzero value of the cosmological constant, still the small value obtained lacks a satisfactory explanation other than just being the result of a mere fine-tuning of the parameters.

Recently new theoretical ideas in extra dimensions have been put forward to attack this problem. In the present talk I shall elaborate on work done in collaboration with Stefan Förste, Zygmunt Lalak and Stéphane Lavignac[3,4,5], where the problem of fine-tuning has been analyzed in the framework of models with extra dimensions that have attracted some attention recently.

One of the most outstanding open problems in quantum field theory is it to find an explanation for the stability of the observed value of the cosmological constant in the presence of radiative corrections. As we will see below (and as has been discussed in several review articles[6,7,8]) a simple quantum field theoretic estimate provides naturally a cosmological constant which is at least 60 orders of magnitude to large. Quantum fluctuations create a vacuum energy which in turn curves the space much stronger than it is observed. Hence, the classical vacuum energy needs to be adjusted in a very accurate way in order to cancel the contributions from quantum effects. This would require a fine-tuning of the fundamental parameters of the theory to an accuracy of at least 60 digits. From the theoretical point of view we consider this as a rather unsatisfactory situation

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and would like to analyze alternatives leading to the observed cosmological constant in a more natural way. In this talk we will focus on brane world scenarios and how they might modify the above mentioned problem. In brane worlds the observed matter is confined to live on a hypersurface of some higher dimensional space, whereas gravity and possibly also some other fields can propagate in all dimensions. This may give some alternative point of view concerning the cosmological constant since the vacuum energy generated by quantum fluctuations of fields living on the brane may not curve the brane itself but instead the space transverse to it. The idea of brane worlds dates back to [9,10,11]. A concrete realization can be found in the context of string theory where matter is naturally confined to live on D-branes [12] or orbifold fixed planes [13]. More recently there has been renewed interest in addressing the problem of the cosmological constant within brane worlds, for an (incomplete) list of references see [14–42] and references therein/thereof.

The talk will be organized as follows. First, we will recall the cosmological constant problem as it appears in ordinary four dimensional quantum field theory. We shall then elaborate on some of the past (four-dimensional) attempts to solve the problem. Subsequently the general set-up of brane worlds will be presented. Particular emphasis will be put on a consistency condition (sometimes also called a sum rule) for warped compactifications that has been overlooked in various attempts to address the problem of the cosmological constant and which is a crucial tool to understand the issue of fine-tunings in the brane world scenario. Then we will study how fine-tunings appear in order to achieve a vanishing cosmological constant in the Randall Sundrum model [14,15]. We shall argue that a similar fine-tuning is needed in the set-up presented in [21,22] once the singularity is resolved. Finally, we elaborate on the issue of the existence of nearby curved solutions and we will argue that it is this questions that has to be addressed if one wants to understand the small value of the cosmological constant.

2 The problem

The observational bound on the cosmological constant is

$$\lambda M_{Pl}^2 \leq 10^{-120} \left(M_{Pl}\right)^4 \tag{1}$$

where M_{Pl} is the Planck mass (of about 10^{19} GeV) and the formula has been written in such a way that the quantity appearing on the left hand side corresponds to the vacuum energy density. This is a very small quantity once one admits the possiblilty of the Planck scale as the fundamantal scale of physics. Even in the particle physics standard model of weak, strong and electromagnetic interactions one would expect a tree level contribution to the vacuum energy of order of several hundred GeV taking into account the scalar potential that leads to electroweak symmetry breaking. Moreover, in quantum field theory we expect additional contributions from perturbative corrections, e.g. at one loop

$$\lambda M_{Pl}^2 = \lambda_0 M_{pl}^2 + \left(\text{UV-cutoff}\right)^4 Str\left(\mathbf{1}\right) \tag{2}$$

in addition to λ_0 the bare (tree level) value of the cosmological constant which can in principle be chosen by hand. The supertrace in (2) is to be taken over degrees of freedom which are light compared to the scale set by the UV-cutoff. Comparison of (1) with (2) shows that one needs to fine-tune 120 digits in $\lambda_0 M_{Pl}^2$ such that it cancels the one-loop contributions with the necessary accuracy. Supersymmetry could ease this problem of radiative corrections (for a review see[43]). If one believes that the world is supersymmetric above the TeV scale one would still need to adjust 60 digits. Instead of adjusting input parameters of the theory to such a high accuracy in order to achieve agreement with observations one would prefer to get (1) as a prediction or at least as a natural result of the theory (in which, for example, only a few digits need to be tuned, if at all).

This is the situation within the framework of four-dimensional quantum field theories. The above discussion might be modified in a brane world setup which we will discuss in this lecture. We should however mention already at this point that "modification" does not necessarily imply an improvement of the situation. Before we get into this discussion let us first recall some attempts to solve the problem in the four-dimensional framework.

3 Possible solutions?

A starting point for a natural solution would be a symmetry that forbids a cosmological constant. In fact, symmetries that could achieve this do exist: e.g. supersymmetry and conformal symmetry. Unfortunately these symmetries are badly broken in nature at a level of at least a few hundred GeV and therefore the problem remains. Still one might think that the presence of such a symmetry would be a first step in the right direction.

A second possible solution could be a dynamical mechanism to relax the cosmological constant. Such a mechanism could be quite similar to the axion mechanism that relaxes the value of the θ parameter in quantum chromodynamics (QCD). This mechanism needs a new ingredient, a propagating field that adjusts is vacuum expectation value dynamically. For a review of these questions see [6,44]. In string theory the so-called "sliding dilaton" could play this role as has been argued in [45,46]. In all these cases, however, one would then expect the existence of an extremely light scalar degree of freedom which would lead to new fifth force that probably should not have escaped our detection.

Other attempts to understand the value of the vacuum energy have used the anthropic principle in one of its various forms. For a review see [6].

Given the present situation it is fair to say that we do not have yet a satisfactory solution of the problem of the cosmological constant, at least in the framework of four-dimensional string and quantum field theories. Could this be better in a higher dimensional world? For an alternative way to address the problem in less than four space-time dimensions see [7].

We should keep in mind, however, that the problem of the cosmological constant is just a problem of fine-tuning the parameters of the theory in a very

special way. We now want to see whether this can be avoided in a higher dimensional set-up.

4 What about extra dimensions $d > 4$?

In the so-called "brane world scenario" matter fields (quarks and leptons, gauge bosons, Higgs bosons) are supposed to be confined to live on a hyper surface (the brane) in a higher dimensional space, whereas gravity and possibly also some additional fields can propagate also in directions transverse to the brane. Such a picture of the universe is motivated by recent developments in (open) string theory [12] and heterotic M-theory [13,47]. Since gravitational interactions are much weaker than the other known interactions, the size of the additional dimension is much less constrained by observations than in usual compactifications. In fact, the size of the additional dimensions might be directly correlated to the strength of four-dimensional gravitational interactions[48]. Looking for example on product compactifications of type I string theory it has been noted that it is possible to push the string scale down to the TeV range when one allows at least two of the compactified dimensions to be "extra large" (i.e. up to a μm)[53].

A first look at the question of the cosmological constant does not look very promising. The naive expectation would be that the cosmological constant in the extra (bulk) dimensions Λ_B and that on the brane, the brane tension T, should vanish separately. We would then essentially have the same situation as in the four-dimensional case, with the additional problem to explain why also Λ_B has to vanish. The known mechanism of a sliding field [45,46] can be carried over to this case [49,50,51,52], but does not shed any new light on the question of the cosmological constant.

A closer inspection of the situation reveals the novel possibility to have a flat brane even in the presence of a nonzero tension T. For a consistent picture, however, here one also has to require a non-zero bulk cosmological constant Λ_B that compensates the vacuum energy (tension) of the brane. In some way this corresponds to a picture where the vacuum energy of the brane does not lead to a curvature on the brane itself, but curves transverse space and leaves the brane flat. Curvature of the brane can flow off to the bulk, a mechanism that is sometimes called "self-tuning".

For such a mechanism to appear we need to consider so-called warped compactifications where brane and transverse space are not just a direct product. We shall see that in this case we can have flat branes embedded in higher dimensional anti de Sitter space, provided certain consistency conditions have been fulfilled.

In the following we will be considering the special case that the brane is 1+3 dimensional and we have one additional direction called y. Then the ansatz for the five dimensional metric is in general ($M, N = 0, \ldots, 4$ and $\mu, \nu = 0, \ldots 3$)

$$ds^2 \equiv G_{MN} dx^M dx^N = e^{2A(y)} \tilde{g}_{\mu\nu} dx^\mu dx^\nu + dy^2 \tag{3}$$

where the brane will be localized at some y. We split

$$\tilde{g}_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \tag{4}$$

into a vacuum value $\bar{g}_{\mu\nu}$ and fluctuations around it $h_{\mu\nu}$. For the vacuum value we will be interested in maximally symmetric spaces, i.e. Minkowski space (M_4), de Sitter space (dS_4), or anti de Sitter space (adS_4). In particular, we chose coordinates such that

$$\bar{g}_{\mu\nu} = \begin{cases} diag\,(-1,1,1,1) & \text{for: } M_4 \\ diag\left(-1, e^{2\sqrt{\bar{\Lambda}}t}, e^{2\sqrt{\bar{\Lambda}}t}, e^{2\sqrt{\bar{\Lambda}}t}\right) & \text{for: } dS_4 \\ diag\left(-e^{2\sqrt{-\bar{\Lambda}}x^3}, e^{2\sqrt{-\bar{\Lambda}}x^3}, e^{2\sqrt{-\bar{\Lambda}}x^3}, 1\right) & \text{for: } adS_4 \end{cases} \tag{5}$$

That means we are looking for 5d spaces which are foliated with maximally symmetric four dimensional slices. Throughout this talk, the five dimensional action will be of the form

$$S_5 = \int d^5x\sqrt{-G}\left[R - \frac{4}{3}\left(\partial\phi\right)^2 - V\left(\phi\right)\right] - \sum_i \int d^5x\sqrt{-g} f_i\left(\phi\right)\delta\left(y-y_i\right). \tag{6}$$

We allow for situations where apart from the graviton also a scalar ϕ propagates in the bulk. The positions of the branes involved are at y_i. With lower case g we denote the induced metric on the brane which for our ansatz is simply

$$g_{\mu\nu} = G_{MN}\delta^M_\mu \delta^N_\nu. \tag{7}$$

The corresponding equations of motion read

$$\begin{aligned}&\sqrt{-G}\left[R_{MN} - \frac{1}{2}G_{MN}R - \frac{4}{3}\partial_M\phi\partial_N\phi + \frac{2}{3}\left(\partial\phi\right)^2 G_{MN} + \frac{1}{2}V\left(\phi\right)G_{MN}\right] \\ &\quad+\frac{1}{2}\sqrt{-g}\sum_i f_i\delta\left(y-y_i\right)g_{\mu\nu}\delta^\mu_M\delta^\nu_N = 0\end{aligned} \quad , \tag{8}$$

$$-\frac{\partial V}{\partial\phi}\sqrt{-G} + \frac{8}{3}\partial_M\left(\sqrt{-G}G^{MN}\partial_N\phi\right) - \sqrt{-g}\sum_i \delta\left(y-y_i\right)\partial_\phi f_i\left(\phi\right) = 0. \tag{9}$$

After integrating over the fifth coordinate in (6) one obtains a four dimensional effective theory. In particular, the gravity part will be of the form

$$S_{4,grav} = M_{Pl}^2 \int d^4x\sqrt{-\tilde{g}}\left(\tilde{R} - \lambda\right), \tag{10}$$

where $\tilde{R}$ is the 4d scalar curvature computed from $\tilde{g}$. The effective Planck mass is $M_{Pl}^2 = \int dy e^{2A}$, (note that we put the five dimensional Planck mass to one). Now, for consistency the ansatz (5) should be a stationary point of (10). This leads to the requirement $\lambda = 6\bar{\Lambda}$. Finally, the on-shell values of the 4d effective

action should be equal to the 5-dimensional one. This results in the consistency condition[4] (see also[24]),

$$\frac{\langle S_5 \rangle}{\int d^4x} = 6\bar{\Lambda} M_{Pl}^2. \tag{11}$$

It has to be fulfilled for all consistent solutions of the Einstein equations, independently whether the branes are flat or curved. Especially for foliations with Poincare invariant slices the vacuum energy $\bar{\Lambda}$ should vanish. Curved solutions would require a corresponding nonzero value of $\bar{\Lambda}$. It is this adjustment of the parameters that replaces the traditional four-dimensional fine-tuning in the brane world picture.

5 A toy example: the Randall-Sundrum set-up

As a warm-up example for a warped compactification we want to study the model presented in [14]. There is no bulk scalar in that model. Therefore, we put $\phi = const$ in (6). Moreover, we plug in

$$V(\phi) = -\Lambda_B \ , \ f_1 = T_1 \ , \ f_2 = T_2, \tag{12}$$

where Λ_B, T_1 and T_2 are constants. There will be two branes: one at $y = 0$ and a second one at $y = y_0$. Denoting with a prime a derivative with respect to y the yy-component of the Einstein equation gives

$$6\left(A'\right)^2 = -\frac{\Lambda_B}{4} \tag{13}$$

Following [14] we are looking for solutions being symmetric under $y \to -y$ and periodic under $y \to y + 2y_0$. The solution to (13) is

$$A = -|y|\sqrt{-\frac{\Lambda_B}{24}}, \tag{14}$$

where $|y|$ denotes the familiar modulus function for $-y_0 < y < y_0$ and the periodic continuation if y is outside that interval. The remaining equation to be solved corresponds to the $\mu\nu$ components of the Einstein equation,

$$3A'' = -\frac{T_1}{4}\delta(y) - \frac{T_2}{4}\delta(y - y_0). \tag{15}$$

This equation is solved automatically by (14) as long as y is neither 0 nor y_0. Integrating equation (15) from $-\epsilon$ to ϵ, relates the brane tension T_1 to the bulk cosmological constant Λ_B,

$$T_1 = \sqrt{-24\Lambda_B}. \tag{16}$$

Integrating around y_0 gives

$$T_2 = -\sqrt{-24\Lambda_B}. \tag{17}$$

These relations arise due to $\bar{\Lambda} = 0$ in the ansatz and can be viewed as fine-tuning conditions for the effective cosmological constant (λ in (10))[16]. Indeed, one finds that the consistency condition (11) is satisfied only when (14) together with both fine-tuning conditions (16) and (17) are imposed. Since the brane tension T_i corresponds to the vacuum energy of matter living in the corresponding brane, the amount of fine-tuning contained in (16), (17) is of the same order as needed in ordinary 4d quantum field theory discussed in the beginning of this talk.

Next we have to address the important question: What happens if the fine-tunings do not hold? Does this necessarily lead to disaster or do solutions exist also in this case. Indeed it has been shown in [16] that in that case solutions exist, however with $\bar{\Lambda} \neq 0$. This closes the argument of interpreting conditions (16) and (17) as fine-tunings of the cosmological constant. It also emphasizes the new problem with the adjustment of the cosmological constant on the brane: how to select the flat solution instead of these "nearby" curved solutions that are continuously connected in parameter (moduli) space.

6 Going back to $\Lambda_B = 0$: does it make sense?

Thus the generic higher dimensional set-up considers nonzero values of brane tensions and the bulk cosmological constant. A fine tuning is needed to arrive at a flat brane with vanishing cosmological constant.

Recently an attempt has been made to study the situation with $\Lambda_B = 0$. We will focus on a " solution" discussed in [21,22] (solution II of the second reference). In this model there is a bulk scalar without a bulk potential

$$V(\phi) = 0. \tag{18}$$

In addition we put one brane at $y = 0$, and a bulk scalar with a very specific coupling to the brane via

$$f_0(\phi) = T_0 e^{b\phi} \quad , \text{ with: } b = \mp\frac{4}{3}. \tag{19}$$

Observe that this model already assumes fine-tuned values Λ_B and b which would have to be explained. We now make the same warped ansatz (3) as before. If again we assume $\bar{\Lambda} = 0$ in (5), the bulk equations seem to be solved by $A' = \pm\frac{1}{3}\phi'$, and

$$\phi(y) = \begin{cases} \pm\frac{3}{4}\log\left|\frac{4}{3}y + c\right| + d \text{ for: } y < 0 \\ \pm\frac{3}{4}\log\left|\frac{4}{3}y - c\right| + d \text{ for: } y > 0 \end{cases}, \tag{20}$$

where d and c are integration constants (they would correspond to the vacuum expectation values of moduli fields in an effective low energy description). Observe that with the logarithm appearing in (20) we are no longer dealing with an exponential warp factor as (3) would suggest. As a result of this we have to worry about possible singularities in the solution under consideration. We shall come back to this point in a moment. Finally, by integrating the equations of motion around $y = 0$ one obtains the matching condition

$$T_0 = 4e^{\pm\frac{4}{3}d}. \tag{21}$$

This means that the matching condition results in an adjustment of an integration constant rather than a model parameter (like in the previously discussed example). So, there seems to be no fine-tuning involved even though we required $\bar{\Lambda} = 0$. As long as one can ensure that contributions to the vacuum energy on the brane couple universely to the bulk scalar as given in (19) it looks as if one can adjust the vev of a modulus such that Poincare invariance on the brane is not broken.

In fact it seems that a miracle has appeared: "solution" (20) is apparently independent of the brane tension T_0. So if one would add something to T_0 on the brane, the solution does not change. This would also solve the problem of potential contributions to the brane tension in perturbation theory, as they can be absorbed in T_0. Is this so-called self-tuning of the vacuum energy a solution to the problem of the cosmological constant? Unfortunately not, since there are some subtleties as we shall discuss now.

We first notice that the uniform coupling of the bulk scalar to any contribution to the vacuum energy on the brane may be problematic due to scaling anomalies in the theory living on the brane [4]. Apart from that one would have to worry about the correct strength of gravitational interactions. In order to be in a agreement with four dimensional gravity, the five dimensional gravitational wave equation should have normalizable zero modes in the given background. In other words this means that the effective four dimensional Planck mass should be finite. For the model considered with a single brane at $y = 0$ and $c < 0$ this implies that $\int_{-\infty}^{\infty} dy e^{2A(y)}$ should be finite. However, plugging in the solution (20) one finds that this is not satisfied. Following [21,22] this could be solved by choosing $c > 0$ and simultaneously cut off the y integration at the singularities at $|y| = \frac{3}{4}c$. This prescription then yields a finite four dimensional Planck mass.

With this choice of parameters, however, we are approaching disaster. Checking the consistency condition (11) one finds that it is not satisfied anymore. The explanation for this is simple – the equation of motions are not satisfied at the singularities, and hence for $c > 0$ **(20) is not a solution to the equations of motion**. It is the singularities that have created the miracle mentioned above.

Of course, it has been often observed that singularities appear in an effective low energy prescription, and that at those points new effects (such as massless particles) appear as a result of an underlying theory to which the effective description breaks down at this point. A celebrated example is $N = 2$ supersymmetric Yang-Mills theory where singularities in the moduli space are due to monopoles or dyons becoming massless at this point [54]. We might then hope that a similar mechanism (e.g. coming from string theory) may save the solution with $c > 0$ and provide a solution to the problem of the cosmological constant. It should be clear by now, that this new physics at the singularity would be the solution of the cosmological constant problem, if such a solution does exist at all.

In the following, we will investigate such a mechanism and see whether it is connected to a potential fine-tuning of the parameters. Does the new physics at the singularity have to know about the actual value of the tension of the brane

at $y = 0$ or does it lead to a relaxation of the cosmological constant independent of T_0?

To start this discussion, we first modify the theory in such a way that we obtain a consistent solution in which the equations of motion are satisfied everywhere. This can, for example, be done by adding two more branes, situated at $|y| = \frac{3}{4}c$ to the setup. We then choose the coupling of the bulk scalar to these branes as follows,

$$f_{\pm}(\phi) = T_{\pm} e^{b_{\pm}\phi}, \tag{22}$$

where the $\pm$ index refers to the brane at $y = \pm\frac{3}{4}c$. These two additional source terms in the action lead to two more matching conditions whose solution is

$$b_{+} = b_{-} = b = \mp\frac{4}{3} \quad \text{and} \quad T_{+} = T_{-} = -\frac{1}{2}T_0. \tag{23}$$

It is obvious that here a third fine tuning (apart from $\Lambda_B = 0$ and $b = \mp\frac{4}{3}$) has to be performed. The amount of fine-tuning implied by these conditions is again determined by the deviation of the vacuum energy on the brane from the observed value. Hence, the situation has worsened with respect to the question of fine-tuning. However, we have learned that the consistency condition (11) is a very important tool to analyze the question of the cosmological constant. A short calculation shows that (23) is essential for the consistency condition (11) to be satisfied.

7 The moduli space of warped solutions

So far, we focused on a very specific model and there remains the question whether this situation is generic. For the given set of parameters we should then scan the available moduli space of solutions parametrized by the values of the bulk cosmological constant Λ_B, the brane tensions T and the various couplings like b of the scalars to the brane. It is quite easy to see that the above observation applies in general (for various explicit examples see [4]). The reason is the fact that the amount of energy carried away from the brane by the bulk scalar needs to be absorbed somewhere else. In principle, it could flow off to infinity, but, as we have seen explicitely in the last chapter, this cannot happen since in this case we would not be able it to localize gravity on the brane. In fact, it has been shown in [27] that localization of gravity is possible only if there is either a fine-tuning between bulk and brane parameters as observed in the original Randall-Sundrum model or there are naked singularities as in the models of [21,22]. For the latter case the consistency condition (11) requires the exactly fine tuned amount of energy from the singularity to match the contribution from the branes. We have seen this explicitly when studying a simple way of "resolving" the singularities by adding new branes with the appropriate tension. However, any other resolution of the singularities will lead to the same conclusions.

So far we have concentrated on solutions that lead to flat branes $\bar{\Lambda} = 0$. A general discussion, however, should also address the question whether the moduli

space of solutions also contains "nearby" curved solutions that are continuously connected to the flat solutions discussed so far. If they exist, the solution of the cosmological constant problem would need to supply arguments why the flat solutions are favoured over these "nearby" curved solutions.

A first step in this direction would be to study the response of the system once the fine-tuning (which appears after the singularities are somehow resolved) is relaxed. In various cases it has been shown that there exist also solutions with $\bar{\Lambda} \neq 0$ [23]. Moreover, for any fixed value of $\bar{\Lambda}$ they fulfill the consistency condition (11) for that value of $\bar{\Lambda}$ [4]. This means, that relaxing the fine-tuning to zero cosmological constant will lead to a consistent (curved) nearby solution with a non-vanishing effective cosmological constant $\bar{\Lambda}$.

It has been argued in the literature that for the specific model which we discussed above ($b = \pm\frac{4}{3}$) there do not exist any nearby curved solutions [21,23]. This would be a rather remarkable result as it would imply that in some way the solution with $\bar{\Lambda} = 0$ would be unique and potentially stable. Observe that the option of a smooth deformation of b away from $|b| = \frac{4}{3}$ is not possible since the $|b| \neq \frac{4}{3}$ solutions are not smoothly connected to the former ones [22]. The above argumention, however, is only true under the asumption that the bulk cosmological constant Λ_B (or the bulk scalar potential) vanishes exactly. The situation in which the scalar field received a nontrivial bulk potential has been studied in [4] with the result that depending on the value of the bulk potential at zero the effective cosmological constant is constrained to a certain non-zero value. Thus also the flat solution with $b = \pm\frac{4}{3}$ is continuously connected to a nearby curved solution with $\bar{\Lambda} \neq 0$ and nonvanishing bulk potential. In all the known cases we thus see that the moduli space does not contain isolated flat solutions. This is another way of stating the fact that the problem of the cosmological constant has not been solved.

So far we have concentrated on the discussion of classical solutions. As we have argued before there is a second aspect of the cosmological constant problem once we consider quantum corrections as well. Generically we would asume that radiative corrections would destroy any fine-tuning of the classical theory if not forbidden by a symmetry. Supersymmetry is an example, but since it is broken in nature at the TeV scale it is not sufficient to stabilize the vacuum energy to the degree of say 10^{-3} eV. The special solution $|b| = \frac{4}{3}$ enjoys an additional symmetry, a variant of scale invariance. This symmetry, however, has a quantum anomaly and therefore cannot survive in the full quantum theory. A way out would be to postulate a model with unbroken scale (or conformal) invariance, i.e. a finite theory with vanishing β-function. But as in the case of supersymmetry we know that this symmetry cannot be valid far below the TeV region and thus cannot be relevant for the stability of the cosmological constant.

8 Outlook

From the above discussion it is clear that the brane world scenario gives a new view on the problem of the cosmological constant. However, the present dis-

cussion has not provided a satisfactory solution, since in all the known cases a fine-tuning is needed to achieve agreement with observations. In fact this fine-tuning is of the same order of magnitude as the one needed in ordinary four dimensional field theory. More work needs to be done to clarify the situation. One direction would be to analyze in detail the possible implications of broken (bulk and brane) supersymmetry in the general set-up. In the four-dimensional case we need broken supersymmetry M_{SUSY} to be somewhere in the TeV region and also the value of the cosmological constant is essentially determined by M_{SUSY}. In the brane world scenario one could hope to separate these scales. Some gymnastics in numerology would suggest $M^2_{\mathrm{SUSY}}/M_{\mathrm{Planck}}$ to be relevant for the (small but nonzero) size of the cosmological constant. Unfortunately we have not yet found a satisfactory model where such a relation is realized and the problem of the size of the cosmological constant still has to wait for a solution.

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References

1. S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], "Measurements of Omega and Lambda from 42 High-Redshift Supernovae," Astrophys. J. **517**, 565 (1999) [astro-ph/9812133].
2. P. M. Garnavich *et al.*, "Constraints on Cosmological Models from Hubble Space Telescope Observations of High-z Supernovae," Astrophys. J. **493**, L53 (1998) [astro-ph/9710123].
3. S. Förste, Z. Lalak, S. Lavignac and H. P. Nilles, "A comment on self-tuning and vanishing cosmological constant in the brane world," Phys. Lett. **B481** (2000) 360 [hep-th/0002164].
4. S. Förste, Z. Lalak, S. Lavignac and H. P. Nilles, "The cosmological constant problem from a brane-world perspective," JHEP **0009** (2000) 034 [hep-th/0006139].
5. S. Forste, "Fine-tuning of the cosmological constant in brane worlds," hep-th/0012029.
6. S. Weinberg, "The Cosmological Constant Problem," Rev. Mod. Phys. **61** (1989) 1.
7. E. Witten, "The cosmological constant from the viewpoint of string theory," hep-ph/0002297.
8. P. Binetruy, "Cosmological constant vs. quintessence," hep-ph/0005037.
9. V. A. Rubakov and M. E. Shaposhnikov, "Extra Space-Time Dimensions: Towards A Solution To The Cosmological Constant Problem," Phys. Lett. **B125** (1983) 139.
10. V. A. Rubakov and M. E. Shaposhnikov, "Do We Live Inside A Domain Wall?," Phys. Lett. **B125** (1983) 136.
11. K. Akama, "An early proposal of 'brane world'," Lect. Notes Phys. **176** (1982) 267 [hep-th/0001113].

12. J. Polchinski, "Dirichlet-Branes and Ramond-Ramond Charges," Phys. Rev. Lett. **75** (1995) 4724 [hep-th/9510017].
13. P. Horava and E. Witten, "Heterotic and type I string dynamics from eleven dimensions," Nucl. Phys. **B460** (1996) 506 [hep-th/9510209].
14. L. Randall and R. Sundrum, "A large mass hierarchy from a small extra dimension," Phys. Rev. Lett. **83** (1999) 3370 [hep-ph/9905221].
15. L. Randall and R. Sundrum, "An alternative to compactification," Phys. Rev. Lett. **83** (1999) 4690 [hep-th/9906064].
16. O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, "Modeling the fifth dimension with scalars and gravity," Phys. Rev. **D62** (2000) 046008 [hep-th/9909134].
17. C. P. Burgess, R. C. Myers and F. Quevedo, "A naturally small cosmological constant on the brane?," Phys. Lett. **B495**, 384 (2000) [hep-th/9911164].
18. A. Kehagias and E. Kiritsis, "Mirage cosmology," JHEP **9911** (1999) 022 [hep-th/9910174].
19. E. Verlinde and H. Verlinde, "RG-flow, gravity and the cosmological constant," JHEP **0005** (2000) 034 [hep-th/9912018].
20. C. Schmidhuber, "AdS(5) and the 4d cosmological constant," Nucl. Phys. **B580** (2000) 140 [hep-th/9912156].
21. N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, "A small cosmological constant from a large extra dimension," Phys. Lett. **B480** (2000) 193 [hep-th/0001197].
22. S. Kachru, M. Schulz and E. Silverstein, "Self-tuning flat domain walls in 5d gravity and string theory," Phys. Rev. **D62** (2000) 045021 [hep-th/0001206].
23. S. Kachru, M. Schulz and E. Silverstein, "Bounds on curved domain walls in 5d gravity," Phys. Rev. **D62** (2000) 085003 [hep-th/0002121].
24. U. Ellwanger, "Constraints on a brane-world from the vanishing of the cosmological constant," Phys. Lett. **B473** (2000) 233 [hep-th/9909103].
25. J. Chen, M. A. Luty and E. Ponton, "A critical cosmological constant from millimeter extra dimensions," JHEP **0009** (2000) 012 [hep-th/0003067].
26. S. P. de Alwis, A. T. Flournoy and N. Irges, "Brane worlds, the cosmological constant and string theory," hep-th/0004125.
27. C. Csaki, J. Erlich, C. Grojean and T. Hollowood, "General properties of the self-tuning domain wall approach to the cosmological constant problem," Nucl. Phys. **B584** (2000) 359 [hep-th/0004133].
28. E. Dudas and J. Mourad, "Brane solutions in strings with broken supersymmetry and dilaton tadpoles," Phys. Lett. **B486** (2000) 172 [hep-th/0004165].
29. G. T. Horowitz, I. Low and A. Zee, "Self-tuning in an outgoing brane wave model," Phys. Rev. **D62** (2000) 086005 [hep-th/0004206].
30. B. Grinstein, D. R. Nolte and W. Skiba, "Adding matter to Poincare invariant branes," Phys. Rev. **D62** (2000) 086006 [hep-th/0005001].
31. V. Barger, T. Han, T. Li, J. D. Lykken and D. Marfatia, "Cosmology and hierarchy in stabilized warped brane models," Phys. Lett. **B488** (2000) 97 [hep-ph/0006275].
32. P. Mayr, "Stringy world branes and exponential hierarchies," JHEP**0011**, 013 (2000) [hep-th/0006204].
33. A. Krause, "A small cosmological constant, grand unification and warped geometry," hep-th/0006226, hep-th/0007233.
34. P. Binetruy, J. M. Cline and C. Grojean, "Dynamical instability of brane solutions with a self-tuning cosmological constant," Phys. Lett. **B489** (2000) 403 [hep-th/0007029].

35. C. Barcelo and M. Visser, "Moduli fields and brane tensions: Generalizing the junction conditions," gr-qc/0008008.
36. Z. Kakushadze, "'Self-tuning' and conformality," Mod. Phys. Lett. **A15** (2000) 1879 [hep-th/0009199].
37. S. Kalyana Rama, "Brane world scenario with m-form field: Stabilisation of radion modulus and self tuning solutions," hep-th/0010121.
38. A. Kehagias and K. Tamvakis, "A self-tuning solution of the cosmological constant problem," hep-th/0011006.
39. J. E. Kim, B. Kyae and H. M. Lee, "A model for self-tuning the cosmological constant," hep-th/0011118.
40. S. B. Giddings and E. Katz, "Effective theories and black hole production in warped compactifications," hep-th/0009176.
41. G. Gibbons, R. Kallosh and A. Linde, "Brane world sum rules," hep-th/0011225.
42. R. Blumenhagen and A. Font, "Dilaton tadpoles, warped geometries and large extra dimensions for non-supersymmetric strings," hep-th/0011269.
43. H. P. Nilles, "Supersymmetry, Supergravity And Particle Physics," Phys. Rept. **110**, 1 (1984).
44. H. P. Nilles, "1997 TASI Lectures on the low-energy limit of string and M-theory," hep-ph/0004064.
45. J. P. Derendinger, L. E. Ibanez and H. P. Nilles, "On The Low-Energy D = 4, N=1 Supergravity Theory Extracted From The D = 10, N=1 Superstring," Phys. Lett. **B155**, 65 (1985).
46. M. Dine, R. Rohm, N. Seiberg and E. Witten, "Gluino Condensation In Superstring Models," Phys. Lett. **B156**, 55 (1985).
47. P. Horava and E. Witten, "Eleven-Dimensional Supergravity on a Manifold with Boundary," Nucl. Phys. **B475**, 94 (1996) [hep-th/9603142].
48. E. Witten, "Strong Coupling Expansion Of Calabi-Yau Compactification," Nucl. Phys. **B471**, 135 (1996) [hep-th/9602070].
49. P. Horava, "Gluino condensation in strongly coupled heterotic string theory," Phys. Rev. D **54**, 7561 (1996) [hep-th/9608019].
50. H. P. Nilles, M. Olechowski and M. Yamaguchi, "Supersymmetry breaking and soft terms in M-theory," Phys. Lett. **B415**, 24 (1997) [hep-th/9707143].
51. H. P. Nilles, M. Olechowski and M. Yamaguchi, "Supersymmetry breakdown at a hidden wall," Nucl. Phys. **B530**, 43 (1998) [hep-th/9801030].
52. A. Lukas, B. A. Ovrut and D. Waldram, "Gaugino condensation in M-theory on S**1/Z(2)," Phys. Rev. D **57**, 7529 (1998) [hep-th/9711197].
53. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, "New dimensions at a millimeter to a Fermi and superstrings at a TeV," Phys. Lett. **B436** (1998) 257 [hep-ph/9804398].
54. N. Seiberg and E. Witten, "Electric - magnetic duality, monopole condensation, and confinement in N=2 supersymmetric Yang-Mills theory," Nucl. Phys. **B426** (1994) 19 [hep-th/9407087].